Statistics

Lecture 12

Feb 19-8:47 AM

$z_{\alpha/2}$ → Critical Value

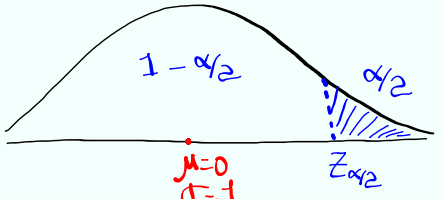
Round to 3-decimal places

It separates the right-tail area of $\alpha/2$ from the rest.

Alpha

$0 < \alpha < 1$

Significance level

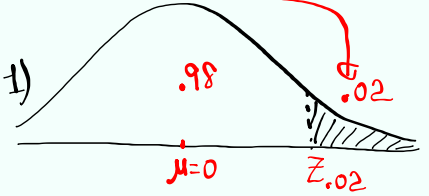


$z_{.02}$

$z_{.02} = \text{invNorm}(.98, 0, 1)$

$= \boxed{2.054}$

$\alpha/2 = .02 \rightarrow \boxed{\alpha = .04}$



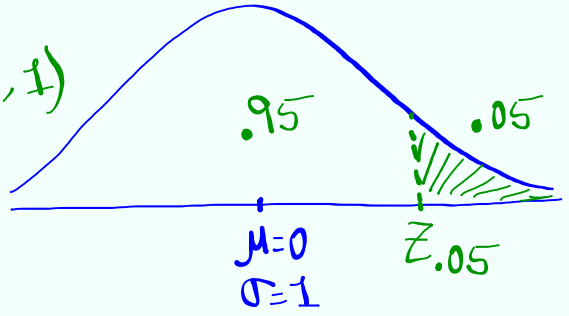
Jul 17-4:33 PM

Find $z_{\alpha/2}$ if $\alpha = .1$

$\alpha/2 = .1/2 = .05$ we want $z_{.05}$

$z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$



Jul 17-4:39 PM

α → Alpha → Significance level

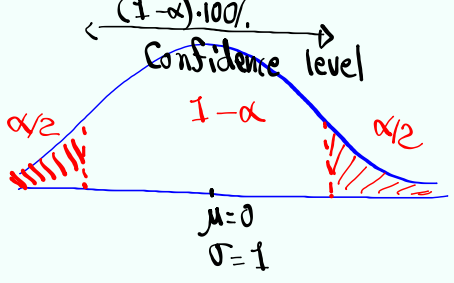
$\alpha/2$ → Area of the right-Tail

$0 < \alpha < 1$

when α not given use .05

$1-\alpha$ → Area in the middle of the graph

$1-\alpha$ in % is called Confidence level



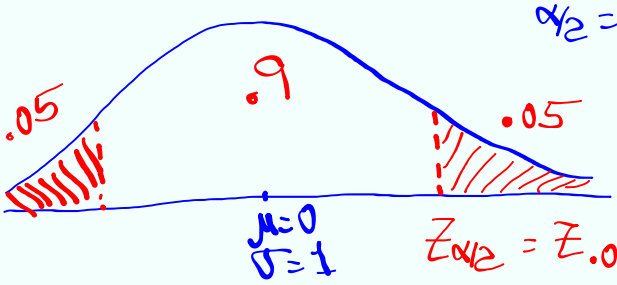
Jul 17-4:42 PM

Find $Z_{\alpha/2}$ for 90% Conf. level.

$1-\alpha = .9$ ← Middle Area

$\alpha = .1$

$\alpha/2 = .05$

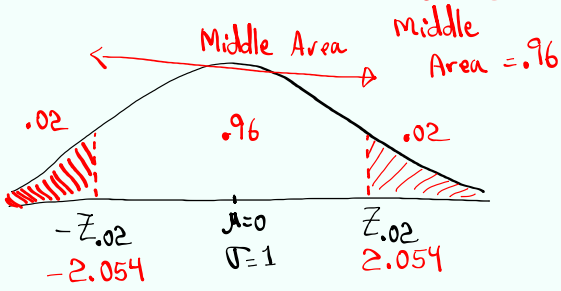


$Z_{\alpha/2} = Z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$

Jul 17-4:46 PM

Find $\pm Z_{\alpha/2}$ for 96% Conf. level.



$Z_{.02} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

α → Significance level

$0 < \alpha < 1$

$1-\alpha$ → Middle Area

$(1-\alpha)\cdot 100\%$ → Confidence level

$\alpha/2$ is the area of the right-Tail.

If α not given ⇒ use .05

If C-level not given ⇒ use 95%

Jul 17-4:50 PM

$t_{\alpha/2}$

T- dist.

Graph is bell-shape, symmetric

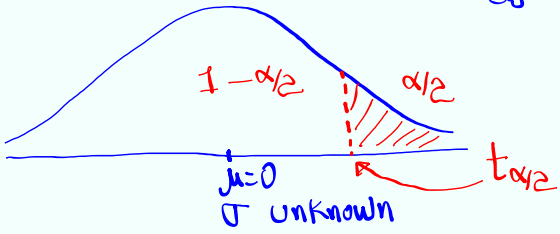
with total area = 1

Mean = Mode = Median

$\mu = 0$, σ is unknown

It comes with degrees of freedom

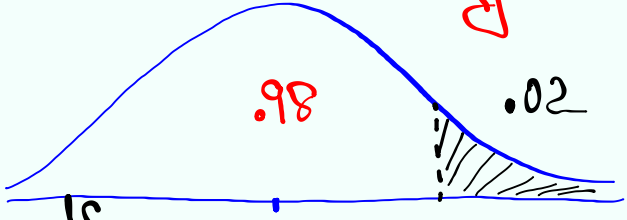
df



use invT to find it.

Jul 17-4:57 PM

Find $t_{.02}$ with df = 9.



invT(LeftArea , df)

invT(.98 , 9)

= 2.398

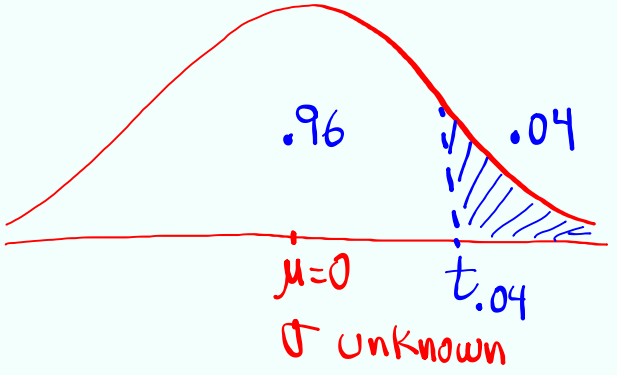
Jul 17-5:01 PM

Find $t_{\alpha/2}$ for $\alpha = .08$ with $df = 14$.

$\alpha/2 = .08/2 = .04$

$t_{.04} = invT(.96, 14)$

$= \boxed{1.887}$

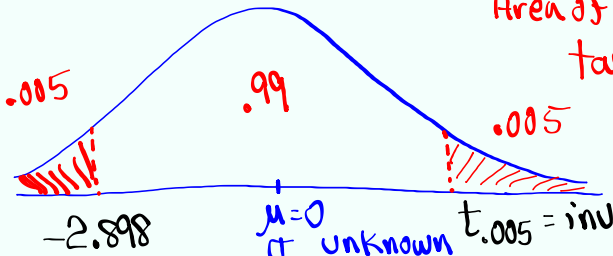


Jul 17-5:04 PM

Find $\pm t_{\alpha/2}$ for 99% Conf. level with $df = 17$.

Middle area .99

Area of each tail $\frac{1-.99}{2} = .005$



$t_{.005} = invT(.995, 17)$

$invT(.005, 17) =$ $= \boxed{2.898}$

If α not given $\Rightarrow$ use .05

If C-level not given $\Rightarrow$ use 95% = .95

Jul 17-5:07 PM

what is degrees of freedom?

How to find it depends on the topic.

I will 24 donuts on Monday.

Stephanie has 24 choices

Nathaniel has 23 choices

⋮ ⋮ ⋮

Weir Bao has 0 choices (1 donut left)

$df = 24 - 1 = 23$

You have 7 clean T-shirts

Monday → 7 choices

Tuesday → 6 choices

Wednesday → 5 choices

⋮

Sunday → 0 choices (1 clean T-shirt)

$df = 7 - 1 = 6$

Jul 17-5:13 PM

Samples ⟷ Statistic

Populations ⟷ Parameters

To learn about Parameters, we must use Statistics.

the values we use from Sample to estimate parameters are called

Point-estimate

To estimate Population		We use Sample	
Proportion	p	Proportion	$\hat{p}$
Mean	μ	Mean	$\bar{x}$
Standard deviation	σ	Standard deviation	s

Jul 17-5:19 PM

Estimating Parameters

Final Answer is a range of values — Confidence Interval

Any estimation comes with Confidence level — Not given, use .95

Estimating Population Proportion

$\_\_\_ < p < \_\_\_$ Conf. interval

$\hat{p} - E < p < \hat{p} + E$

$\hat{p}$: Sample Proportion ↔ Point-estimate

E: Margin of Error

$\hat{p} = \frac{x}{n}$ ← # of favorable responses; ← Sample Size

$\hat{q} = 1 - \hat{p}$

$x = n\hat{p}$ if decimal ⇒ Always round up to a whole #

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$z_{\alpha/2}$: Critical Value for $(1-\alpha)\cdot 100\%$ Conf. level.

Jul 17-5:40 PM

I surveyed 400 students and 300 of them had iPhone. $n = 400$, $x = 300$

$\hat{p} = \frac{x}{n} = \frac{300}{400} = .75$ $\quad \hat{q} = .25$

I want to find conf. interval for Prop. of all students that have iphone.

Use C-level 90%.

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

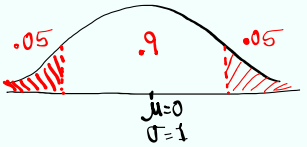
$= 1.645 \cdot \sqrt{\frac{(.75)(.25)}{400}} \approx .04$

$< p <$

$\hat{p} - E < p < \hat{p} + E$

$.75 - .04 < p < .75 + .04$

$.71 < p < .79$



We are 90% Confident that between 71% & 79% of all students have iPhone.

$z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = 1.645$

STAT ▶ TESTS [1] 1-PropZInt

$x = 300$

$n = 400$

C-level: .9

Calculate

$.714 < p < .7856\ldots$

$.71 < p < .79$

Jul 17-5:48 PM

I surveyed 250 voters, and I asked them if they have decided already who to vote for. 100 of them said Yes.

$n = 250$ $x = 100$

Find 98% Conf. interval for the prop. of all voters that have already decided on voting for. C-level: .98

STAT → TESTS ↓ A:1-PropZInt

$x = 100$
$n = 250$
C-level: .98
Calculate

$E = \frac{.47 - .33}{2} = .07$

$\hat{p} = \frac{.47 + .33}{2} = .4$

$.33 < p < .47$

We are 98% Confident that between 33% & 47% of all voters have already decided

Jul 17-6:01 PM

The College surveyed 285 students and 36% of them were in favor of online classes.

$n = 285$
$\hat{p} = .36$ $\Rightarrow x = n\hat{p} = 285(.36) = 103$
NO C-level ⇒ use .95

Find Conf. interval for the prop. of all Students that are in favor of online classes.

1-PropZInt

$E = \frac{.42 - .31}{2} = .055$

$\hat{p} = \frac{.42 + .31}{2} = .365$

$.31 < p < .42$

We are 95% Confident that between 31% & 42% of all Students are in favor of online classes.

Jul 17-6:10 PM

Estimating Population Mean:

$$\_ < \mu < \_$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$\bar{x}$: Sample Mean ⇔ Point-estimate

E: Margin of error

Case I: σ Known	Case II: σ unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $df = n-1$
TI: Z Interval inpt: Stats	TI: T Interval inpt: STATS
$E = \frac{-}{2}$	$\bar{x} = \frac{+}{2}$

Jul 17-6:24 PM

Given $n = 32$, $\bar{x} = 88$, $\sigma = 10$

C-level: .99

Find Conf. interval for μ.

Z Interval σ known

T Interval

$$E = \frac{93 - 83}{2} = 5$$

$$\bar{x} = \frac{93 + 83}{2} = 88$$

inpt: Stats

$\sigma = 10$

$\bar{x} = 88$ ← Point-estimate is a whole #

$n = 32$

C-level: .99

$83 < \mu < 93$

Jul 17-6:31 PM

Given $n=16$, $\bar{x}=124.8$, $S=12.5$

$df=15$

Find Conf. interval for μ.

Z Interval

T Interval | σ unknown

inpt: STATS

$\bar{x}=124.8$ ← Point-estimate is 1-dec.

$S=12.5$

$n=16$

C-level: .95

Calculate

$118.1 < \mu < 131.5$

$E = \frac{131.5 - 118.1}{2} = 6.7$

$\bar{x} = \frac{131.5 + 118.1}{2} = 124.8$

Jul 17-6:36 PM

I randomly selected 35 bank tellers, their mean age was 28 yrs.

$n=35$

$\bar{x}=28$

It is known that standard deviation of ages of all bank tellers is 7.5 yrs.

$\sigma=7.5$

C-level: .99

Find 99% Conf. interval for the mean age of all bank tellers.

$25 < \mu < 31$

Z Interval | σ known

T Interval

inpt: Stats

$\sigma: 7.5$

$\bar{x}=28$

$n=35$

C-level: .99

Jul 17-6:42 PM

I randomly selected 10 exams, here are the scores

75 82 100 80
90 95 70 100
68 88

Find

$\bar{x} = 85$

$s = 12$

} Round to whole

Find 90% Conf. interval for the mean score of all exams.

inpt: Stats

$\bar{x} = 85$
$s = 12$
$n = 10$
C-level: .9

$78 < \mu < 92$

$E = \frac{92-78}{2} = 7$

$\bar{x} = \frac{92+78}{2} = 85$

ZInterval

TInterval σ Unknown

Jul 17-6:50 PM